Primer on Inventory Management – Solutions

Newsvendor Model

Solution to Exercise 1:

Solution:

i. Expected overage quantity:

$$\begin{aligned}
E\left[S-y\right]^{+} &= \sum_{y=0}^{S}(S-y)\cdot p_y \\
&= 4\cdot 0.05 + 3\cdot 0.1 + 2\cdot 0.2 + 1\cdot 0.3 + 0\cdot 0.2 \\
&= 1.2
\end{aligned}$$

ii. Expected underage quantity;

$$\begin{aligned}
E\left[y-S\right]^{+} &= \sum_{y=S}^{\infty}(y-S)\cdot p_y \\
&= 0\cdot 0.2 + 1\cdot 0.1 + 2\cdot 0.05 \\
&= 0.2
\end{aligned}$$

In some cases, the infinite sum is cumbersome to compute. An alternative way is:

$$\begin{aligned}
E\left[y-S\right]^{+} &= \sum_{y=S}^{\infty}\left(y-S\right)\cdot p_y \\
&= \sum_{y=0}^{\infty}\left(y-S\right)\cdot p_y - \sum_{y=0}^{S}\left(y-S\right)\cdot p_y \\
&= \sum_{y=0}^{\infty} y\cdot p_y - \sum_{y=0}^{\infty} S\cdot p_y - \sum_{y=0}^{S}\left(y-S\right)\cdot p_y \\
&= E\left(y\right) - S\cdot\sum_{y=0}^{\infty} p_y - \sum_{y=0}^{S}\left(y-S\right)\cdot p_y \\
&= E\left(y\right) - S\cdot 1 + \sum_{y=0}^{S}\left(S-y\right)\cdot p_y \\
&= \mu - S + E\left[S-y\right]^{+} \\
&= 3-4+1.2 \\
&= 0.2
\end{aligned}$$

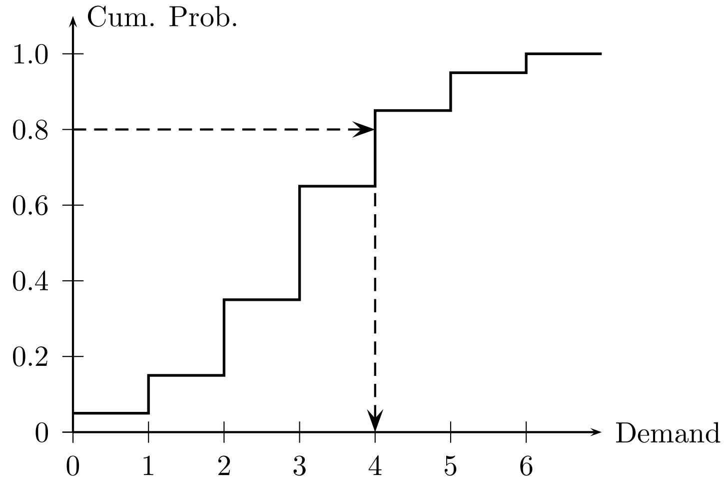
iii. Overage cost: $c_o = c - v = 1.00$ EUR $- 0.50$ EUR $= 0.50$ EUR
Underage cost: $c_u = r - c = 3.00$ EUR $- 1.00$ EUR $= 2.00$ EUR

iv.

$$\begin{aligned}
Z\left(S\right) &= c_o\cdot E\left[S-y\right]^{+} + c_u\cdot E\left[y-S\right]^{+} \\
&= 0.50\cdot 1.2 + 2.00\cdot 0.2 \\
&= 1.00
\end{aligned}$$

v. The Critical Ratio reads

$$\mathrm{CR} = \frac{c_u}{c_u + c_o} = \frac{2}{2+0.5} = 0.8$$



Observe that $S^\star = 4$. The expected cost are the same as computed in iii.

Solution to Exercise 2:

i. $c_u = 25,000 - 12,000 = 13,000$ and $c_o = 12,000 - 8,000 = 4,000$. Therefore,

$$\text{CR} = \frac{c_u}{c_u + c_o} = \frac{13,000}{13,000 + 4,000} = \frac{13}{17} \approx 0.76$$

The distribution function of a poisson distribution with rate μ is

$$p_y = \frac{\mu^y}{y!} \cdot e^{-\mu}$$

For a poisson distribution with rate $\mu = 3$ we have

y	p_y	P_y
0	0.050	0.050
1	0.149	0.199
2	0.224	0.423
3	0.224	0.647
4	0.168	0.815
5	0.101	0.916
6	0.050	0.966
7	0.022	0.988
8	0.008	0.996
9	0.003	0.999
10	0.001	1.000

The optimal order quantity is given by the smallest y for wich the cumulative distribution function of the demand is greater or equal to the critical ratio. In our case, this yields $S^\star = 4$.

ii. Expected overage quantity:

$$
\begin{aligned}
E\left[S-y\right]^{+} &= \sum_{y=0}^{S} (S-y)\cdot p_y \\
&= 4\cdot 0.050 + 3\cdot 0.149 + 2\cdot 0.224 + 1\cdot 0.224 \\
&= 1.319
\end{aligned}
$$

The expected number of spare parts left over after three years is 1.319.

iii. Expected underage quantity:

$$
\begin{aligned}
E\left[y-S\right]^{+} &= \sum_{y=S}^{\infty} (y-S)\cdot p_y \\
&= \mu - S - \sum_{y=0}^{S} (y-S)\cdot p_y \\
&= \mu - S + E\left[S-y\right]^{+} \\
&= 3 - 4 + 1.319 \\
&= 0.319
\end{aligned}
$$

The expected number of spare parts to be custom made in three years is 0.319.

iv. Expected cost:

$$\begin{aligned}
Z(S) &= c_o \cdot E[S-y]^+ + c_u \cdot E[y-S]^+ \\
&= 4,000 \cdot 1.319 + 13,000 \cdot 0.319 \\
&= 9,423
\end{aligned}$$

Expected total cost is therefore

$$\begin{aligned}
\Pi(S) &= c \cdot \mu + Z(S) \\
&= 12,000 \cdot 3 - 9,423 \\
&= 45,423.
\end{aligned}$$

Solution to Exercise 3:

i. We first determine underage and overage cost. Note, that the imputed entrepreneural salary of 10,000 EUR and the rent of 2,000 EUR are not relevant for the seller's decision. Only purchasing and transportation cost are relevant. Hence, $c_o = c - v = 10 + 2 - 1 = 11$. In case of a shortage, the seller will always buy a tree for 62 EUR from his competitor. However, for the detemrination of underage cost, we only have to account for the additional cost he occurs, i.e. $c_u = 62 - 12 = 50$. Subsequently, we obtain a critical ratio of

$$\frac{c_u}{c_u + c_o} = \frac{50}{50 + 11} \approx 0.8197$$

With

$$z = F^{-1}(CR) = F^{-1}(0.8197) \approx 0.914$$

we obtain

$$S^\star = \mu + z \cdot \sigma = 10,510 + 0.914 \cdot 2,114 = 12,442.$$

ii. Under Normally distributed demand the expected cost can be computed as

$$\begin{aligned}
Z(S^\star) &= (c_u + c_o) \cdot f_{N(0,1)}(z) \cdot \sigma \\
&= (50 + 11) \cdot 0.263 \cdot 2,114 \\
&= 33,914.
\end{aligned}$$

iii. The expected profit can be computed as

$$\begin{aligned}
\Pi(S^\star) &= (r - c) \cdot \mu - Z(S^\star) \\
&= (30 - 12) \cdot 10,510 - 33,914 \\
&= 155,266.
\end{aligned}$$