Primer on Inventory Management – Solutions

Economic Order Quantity Model

Solution to Exercise 1:

i.

$$\frac{\partial}{\partial x} Z(x) = \frac{h}{2} - K\frac{\mu}{x^2}$$
$$\frac{\partial^2}{\partial x^2} Z(x) = \frac{2K\mu}{x^3} \quad > 0 \quad \forall x > 0;$$

ii.

$$\begin{aligned} & \frac{\partial}{\partial x} Z(x) && = 0 \\ \Leftrightarrow \quad & \frac{h}{2} - \frac{K\mu}{x^2} && = 0 \\ \Leftrightarrow \quad & x^2 && = \frac{2K\mu}{h} \\ \Leftrightarrow \quad & x^\star && = \pm\sqrt{\frac{2K\mu}{h}} \end{aligned}$$

Since x cannot be negative, only the positive solution makes sense.

iii. We know that

$$Z(x) = h\frac{x}{2} + K\frac{\mu}{x} + c\mu \tag{1}$$

and

$$x^\star = \sqrt{\frac{2K\mu}{h}} \tag{2}$$

Instertion of (2) in (1) yields

$$Z(x^\star) = c \cdot \mu + \sqrt{2K\mu h} \tag{3}$$

iv. In our cost function the inventory holding cost is

$$h\frac{x}{2} \tag{4}$$

and the fixed order cost is

$$K\frac{\mu}{x} \tag{5}$$

The optimal order quantity is

$$x^{\star} = \sqrt{\frac{2K\mu}{h}} \tag{6}$$

Instertion of (6) in (4) and (5) yields

$$h\frac{x^{\star}}{2} = \frac{h}{2} \cdot \sqrt{\frac{2K\mu}{h}} = \sqrt{\frac{K\mu h}{2}} \tag{7}$$

and

$$K\frac{\mu}{x^{\star}} = \frac{K\mu}{\sqrt{\frac{2K\mu}{h}}} = K\mu \cdot \sqrt{\frac{2K\mu}{h}} = \sqrt{\frac{K\mu h}{2}} \tag{8}$$

Obviously, (7) and (8) are equal.

Solution to Exercise 2:

First, determine the annual demand rate:

$$\mu = 60 \text{ units/week} \cdot 52 \text{ weeks/year} = 3,120 \text{ units/year}$$

The annual holding cost rate is:

$$h = 0.25 \text{ year}^{-1} \cdot 0.02 \text{ EUR/unit} = 0.005 \text{ EUR/(unit} \cdot \text{year}$$

This yields:

$$x^{\star} = \sqrt{\frac{2K\mu}{h}} = \sqrt{\frac{2 \cdot 12 \cdot 3,120}{0.005}} = 3,870 \text{ units.}$$

The cost of the optimal solution is:

$$\begin{aligned} Z(x^{\star}) &= c \cdot \mu + \sqrt{2K\mu h} \\ &= 0.02 \cdot 3,120 + \sqrt{2 \cdot 12 \cdot 3,120 \cdot 0.005} \\ &= 81.75 \text{ EUR} \end{aligned}$$

Solution to Exercise 3:

i.

$$x^\star = \sqrt{\frac{2K\mu}{h}} = \sqrt{\frac{2 \cdot 40 \cdot 10.6}{0.1}} = 92.09 \approx 92$$

ii.

$$\begin{aligned} Z\left(x^\star\right) &= c \cdot \mu + \sqrt{2K\mu h} \\ &= 1 \cdot 10.6 + \sqrt{2 \cdot 40 \cdot 10.6 \cdot 0.1} \\ &= 19.81 \text{ EUR} \end{aligned}$$

iii. For $x' = 0.9 \cdot x^\star = 0.9 \cdot 92 = 82.8 \approx 83$ we compute

$$\begin{aligned} Z\left(83\right) &= c \cdot \mu + h\frac{83}{2} + K\frac{\mu}{83} \\ &= 1 \cdot 10.6 + 0.1\frac{83}{2} + 40\frac{10.6}{83} \\ &= 19.86 \end{aligned}$$

Observe, that the difference to $Z\left(x^\star\right)$ is very small!

iv. We know that

$$\mathcal{Z}\left(x\right) = h\frac{x}{2} + K\frac{\mu}{x}$$

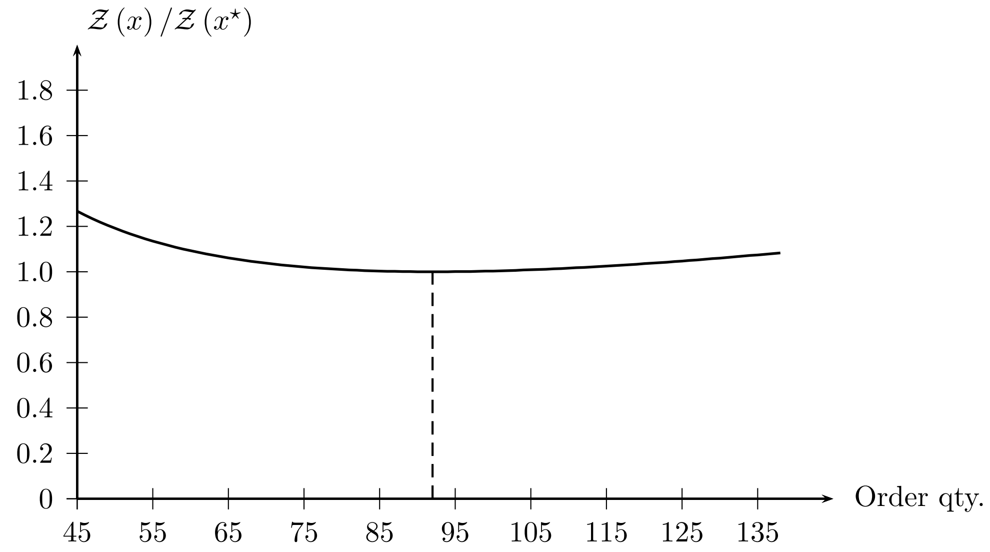
and

$$\mathcal{Z}\left(x^\star\right) = \sqrt{2K\mu h}$$

Subsequently,

$$\begin{aligned} \frac{\mathcal{Z}\left(x\right)}{\mathcal{Z}\left(x^\star\right)} &= \frac{h\frac{x}{2} + K\frac{\mu}{x}}{\sqrt{2K\mu h}} \\ &= \frac{x}{2} \cdot \frac{h}{\sqrt{2K\mu h}} + \frac{1}{x} \cdot \frac{K\mu}{\sqrt{2K\mu h}} \\ &= \frac{x}{2} \cdot \frac{1}{x^\star} + \frac{1}{2x} \cdot \frac{x^\star}{1} \\ &= \frac{1}{2}\left(\frac{x}{x^\star} + \frac{x^\star}{x}\right) \end{aligned}$$

v. The sensitivity plot reads:



vi. The solution is remarkably insensitive.