



Universität zu Köln

Seminar für Allgemeine Betriebswirtschaftslehre, Supply Chain Management und Management Science

Primer on Inventory Management

Newsvendor Model

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News vendor model

Assumptions

- Orders are placed before demand is known
- Demand distribution is known when order is placed
- Unsold newspapers can be salvaged
- Excess demand is lost
- The objective is maximizing expected profit

Parameters

- p_y Probability that the demand will be y
- $f(y)$ Density function of demand y
- $F(y)$ Distribution function of demand y
- c Unit order cost (2 €/unit)
- r Unit revenue (5 €/unit)
- v Unit salvage value (1 €/unit)

Decision variable

S Order quantity (2 units)

Expected overage and expected underage quantities

Demand	Probability	Overage	Underage		
y	p_y	$[S - y]$	$[y - S]$	$[S - y] p_y$	$[y - S] p_y$
0	0.10	2		$2 \cdot 0.10 = 0.2$	
1	0.30	1		$1 \cdot 0.30 = 0.3$	
$S = 2$	0.30	0	0	$0 \cdot 0.30 = 0$	$0.30 \cdot 0 = 0$
3	0.20		1		$0.20 \cdot 1 = 0.2$
4	0.10		2		$0.10 \cdot 2 = 0.2$
				0.5	0.4
				E[Overage]	E[Underage]

Unit overage cost and unit underage cost

c_o Unit overage cost = $c - v = 2 \text{ €/unit} - 1 \text{ €/unit} = 1 \text{ €/unit}$

c_u Unit underage cost = $r - c = 5 \text{ €/unit} - 2 \text{ €/unit} = 3 \text{ €/unit}$

Expected cost

$$Z(S) = c_o E[\text{Overage}] + c_u E[\text{Underage}]$$

$$= c_o \sum_{y=0}^S (S - y) p_y + c_u \sum_{y=S}^{\infty} (y - S) p_y$$

$$Z(2) = 1 \text{ €/unit} \cdot 0.5 \text{ units} + 3 \text{ €/unit} \cdot 0.4 \text{ units} = 1.70 \text{ €}$$

Optimal order quantity

Objective function

$$Z(S) = c_o \sum_{y=0}^S (S-y)p_y + c_u \sum_{y=S}^{\infty} (y-S)p_y$$

$$Z(S) = c_o \int_{y=0}^S (S-y)f(y)dy + c_u \int_S^{\infty} (y-S)f(y)dy$$

$$\frac{d}{dS} Z(S) = c_o F(S) - c_u (1-F(S))$$

$$\frac{d^2}{dS^2} Z(S) = c_o f(S) + c_u f(S) \geq 0$$

First order condition

$$\frac{d}{dS} Z(S) = 0$$

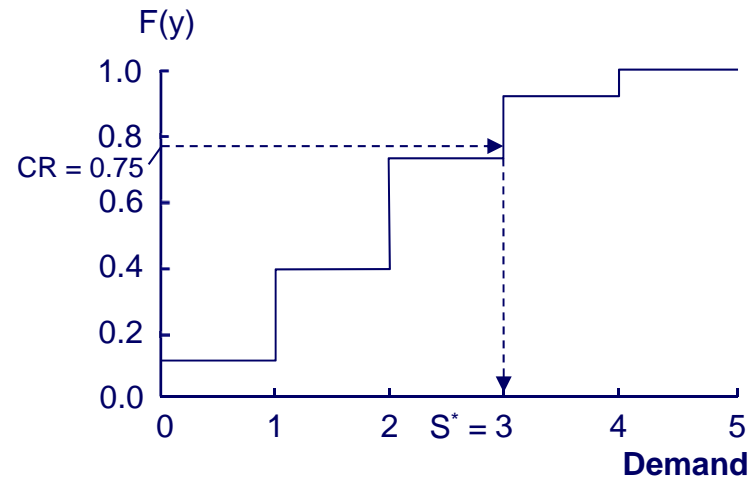
$$c_o F(S) - c_u (1-F(S)) = 0$$

$$F(S) = \frac{c_u}{c_u + c_o} = \text{CR (Critical Ratio)}$$

$$S^* = F^{-1}\left(\frac{c_u}{c_u + c_o}\right)$$

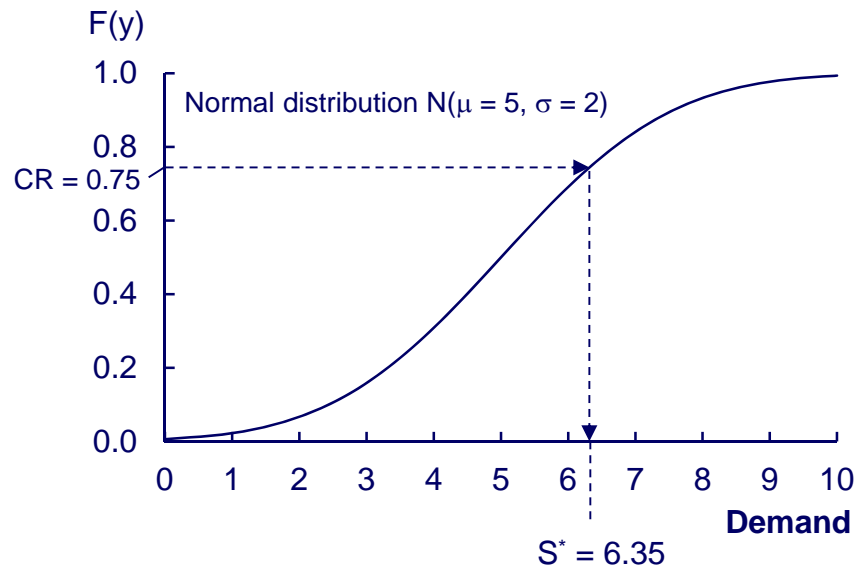
Example

$$\text{CR} = \frac{c_u}{c_u + c_o} = \frac{3}{3+1} = 0.75$$



$$\begin{aligned} Z(S^*) &= c_o \sum_{y=0}^3 (S-y)p_y + c_u \sum_{y=3}^4 (y-S)p_y \\ &= 1 \text{ €/unit} \cdot [(3-0) \cdot 0.10 + (3-1) \cdot 0.30 + (3-2) \cdot 0.30 + (3-3) \cdot 0.20] \\ &\quad + 3 \text{ €/unit} \cdot [(3-3) \cdot 0.20 + (4-3) \cdot 0.10] \\ &= 1.50 \text{ €} \end{aligned}$$

Optimal solution under Normally distributed demand



Optimal order quantity

$$S^* = \mu + z\sigma \quad z = F_{N(0,1)}^{-1} \left(\frac{c_u}{c_u + c_o} \right)$$

From Normal distribution table or from Excel `NORM.INV(0.75,0,1)`:

$$z = F_{N(0,1)}^{-1}(0.75) = 0.674$$

$$S^* = 5 + 0.674 \cdot 2 = 6.35$$

Optimal expected cost

$$Z(S^*) = (c_u + c_o) \cdot f_{N(0,1)}(z) \cdot \sigma$$

From Normal distribution table or from Excel `NORM.VERT(0.674,0,1,0)`

$$f_{N(0,1)}(0.674) = 0.318$$

$$Z(6.35) = (3 \text{ €/unit} + 1 \text{ €/unit}) \cdot 0.318 \cdot 2 \text{ units} = 2.54 \text{ €}$$

Optimal expected profit

$$\Pi(S^*) = (r - c)\mu - Z(S^*)$$

$$\Pi(S^*) = (5 \text{ €/unit} - 1 \text{ €/unit})5 \text{ units} - 2.54 \text{ €} = 17.46 \text{ €}$$

Appendix

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Table of the Standard Normal Distribution

z	f(z)	F(z)	L(z)	z	f(z)	F(z)	L(z)	z	f(z)	F(z)	L(z)	z	f(z)	F(z)	L(z)
-3.00	0.0044	0.0013	3.0004	-2.50	0.0175	0.0062	2.5020	-2.00	0.0540	0.0228	2.0085	-1.50	0.1295	0.0668	1.5293
-2.99	0.0046	0.0014	2.9904	-2.49	0.0180	0.0064	2.4921	-1.99	0.0551	0.0233	1.9987	-1.49	0.1315	0.0681	1.5200
-2.98	0.0047	0.0014	2.9804	-2.48	0.0184	0.0066	2.4821	-1.98	0.0562	0.0239	1.9890	-1.48	0.1334	0.0694	1.5107
-2.97	0.0048	0.0015	2.9704	-2.47	0.0189	0.0068	2.4722	-1.97	0.0573	0.0244	1.9792	-1.47	0.1354	0.0708	1.5014
-2.96	0.0050	0.0015	2.9604	-2.46	0.0194	0.0069	2.4623	-1.96	0.0584	0.0250	1.9694	-1.46	0.1374	0.0721	1.4921
-2.95	0.0051	0.0016	2.9505	-2.45	0.0198	0.0071	2.4523	-1.95	0.0596	0.0256	1.9597	-1.45	0.1394	0.0735	1.4828
-2.94	0.0053	0.0016	2.9405	-2.44	0.0203	0.0073	2.4424	-1.94	0.0608	0.0262	1.9500	-1.44	0.1415	0.0749	1.4736
-2.93	0.0055	0.0017	2.9305	-2.43	0.0208	0.0075	2.4325	-1.93	0.0620	0.0268	1.9402	-1.43	0.1435	0.0764	1.4643
-2.92	0.0056	0.0018	2.9205	-2.42	0.0213	0.0078	2.4226	-1.92	0.0632	0.0274	1.9305	-1.42	0.1456	0.0778	1.4551
-2.91	0.0058	0.0018	2.9105	-2.41	0.0219	0.0080	2.4126	-1.91	0.0644	0.0281	1.9208	-1.41	0.1476	0.0793	1.4459
-2.90	0.0060	0.0019	2.9005	-2.40	0.0224	0.0082	2.4027	-1.90	0.0656	0.0287	1.9111	-1.40	0.1497	0.0808	1.4367
-2.89	0.0061	0.0019	2.8906	-2.39	0.0229	0.0084	2.3928	-1.89	0.0669	0.0294	1.9013	-1.39	0.1518	0.0823	1.4275
-2.88	0.0063	0.0020	2.8806	-2.38	0.0235	0.0087	2.3829	-1.88	0.0681	0.0301	1.8916	-1.38	0.1539	0.0838	1.4183
-2.87	0.0065	0.0021	2.8706	-2.37	0.0241	0.0089	2.3730	-1.87	0.0694	0.0307	1.8819	-1.37	0.1561	0.0853	1.4092
-2.86	0.0067	0.0021	2.8606	-2.36	0.0246	0.0091	2.3631	-1.86	0.0707	0.0314	1.8723	-1.36	0.1582	0.0869	1.4000
-2.85	0.0069	0.0022	2.8506	-2.35	0.0252	0.0094	2.3532	-1.85	0.0721	0.0322	1.8626	-1.35	0.1604	0.0885	1.3909
-2.84	0.0071	0.0023	2.8407	-2.34	0.0258	0.0096	2.3433	-1.84	0.0734	0.0329	1.8529	-1.34	0.1626	0.0901	1.3818
-2.83	0.0073	0.0023	2.8307	-2.33	0.0264	0.0099	2.3334	-1.83	0.0748	0.0336	1.8432	-1.33	0.1647	0.0918	1.3727
-2.82	0.0075	0.0024	2.8207	-2.32	0.0270	0.0102	2.3235	-1.82	0.0761	0.0344	1.8336	-1.32	0.1669	0.0934	1.3636
-2.81	0.0077	0.0025	2.8107	-2.31	0.0277	0.0104	2.3136	-1.81	0.0775	0.0351	1.8239	-1.31	0.1691	0.0951	1.3546
-2.80	0.0079	0.0026	2.8008	-2.30	0.0283	0.0107	2.3037	-1.80	0.0790	0.0359	1.8143	-1.30	0.1714	0.0968	1.3455
-2.79	0.0081	0.0026	2.7908	-2.29	0.0290	0.0110	2.2938	-1.79	0.0804	0.0367	1.8046	-1.29	0.1736	0.0985	1.3365
-2.78	0.0084	0.0027	2.7808	-2.28	0.0297	0.0113	2.2839	-1.78	0.0818	0.0375	1.7950	-1.28	0.1758	0.1003	1.3275
-2.77	0.0086	0.0028	2.7708	-2.27	0.0303	0.0116	2.2740	-1.77	0.0833	0.0384	1.7854	-1.27	0.1781	0.1020	1.3185
-2.76	0.0088	0.0029	2.7609	-2.26	0.0310	0.0119	2.2641	-1.76	0.0848	0.0392	1.7758	-1.26	0.1804	0.1038	1.3095
-2.75	0.0091	0.0030	2.7509	-2.25	0.0317	0.0122	2.2542	-1.75	0.0863	0.0401	1.7662	-1.25	0.1826	0.1056	1.3006
-2.74	0.0093	0.0031	2.7409	-2.24	0.0325	0.0125	2.2444	-1.74	0.0878	0.0409	1.7566	-1.24	0.1849	0.1075	1.2917
-2.73	0.0096	0.0032	2.7310	-2.23	0.0332	0.0129	2.2345	-1.73	0.0893	0.0418	1.7470	-1.23	0.1872	0.1093	1.2827
-2.72	0.0099	0.0033	2.7210	-2.22	0.0339	0.0132	2.2246	-1.72	0.0909	0.0427	1.7374	-1.22	0.1895	0.1112	1.2738
-2.71	0.0101	0.0034	2.7110	-2.21	0.0347	0.0136	2.2147	-1.71	0.0925	0.0436	1.7278	-1.21	0.1919	0.1131	1.2650
-2.70	0.0104	0.0035	2.7011	-2.20	0.0355	0.0139	2.2049	-1.70	0.0940	0.0446	1.7183	-1.20	0.1942	0.1151	1.2561
-2.69	0.0107	0.0036	2.6911	-2.19	0.0363	0.0143	2.1950	-1.69	0.0957	0.0455	1.7087	-1.19	0.1965	0.1170	1.2473
-2.68	0.0110	0.0037	2.6811	-2.18	0.0371	0.0146	2.1852	-1.68	0.0973	0.0465	1.6992	-1.18	0.1989	0.1190	1.2384
-2.67	0.0113	0.0038	2.6712	-2.17	0.0379	0.0150	2.1753	-1.67	0.0989	0.0475	1.6897	-1.17	0.2012	0.1210	1.2296
-2.66	0.0116	0.0039	2.6612	-2.16	0.0387	0.0154	2.1655	-1.66	0.1006	0.0485	1.6801	-1.16	0.2036	0.1230	1.2209
-2.65	0.0119	0.0040	2.6512	-2.15	0.0396	0.0158	2.1556	-1.65	0.1023	0.0495	1.6706	-1.15	0.2059	0.1251	1.2121
-2.64	0.0122	0.0041	2.6413	-2.14	0.0404	0.0162	2.1458	-1.64	0.1040	0.0505	1.6611	-1.14	0.2083	0.1271	1.2034
-2.63	0.0126	0.0043	2.6313	-2.13	0.0413	0.0166	2.1360	-1.63	0.1057	0.0516	1.6516	-1.13	0.2107	0.1292	1.1946
-2.62	0.0129	0.0044	2.6214	-2.12	0.0422	0.0170	2.1261	-1.62	0.1074	0.0526	1.6422	-1.12	0.2131	0.1314	1.1859
-2.61	0.0132	0.0045	2.6114	-2.11	0.0431	0.0174	2.1163	-1.61	0.1092	0.0537	1.6327	-1.11	0.2155	0.1335	1.1773
-2.60	0.0136	0.0047	2.6015	-2.10	0.0440	0.0179	2.1065	-1.60	0.1109	0.0548	1.6232	-1.10	0.2179	0.1357	1.1686
-2.59	0.0139	0.0048	2.5915	-2.09	0.0449	0.0183	2.0966	-1.59	0.1127	0.0559	1.6138	-1.09	0.2203	0.1379	1.1600
-2.58	0.0143	0.0049	2.5816	-2.08	0.0459	0.0188	2.0868	-1.58	0.1145	0.0571	1.6044	-1.08	0.2227	0.1401	1.1514
-2.57	0.0147	0.0051	2.5716	-2.07	0.0468	0.0192	2.0770	-1.57	0.1163	0.0582	1.5949	-1.07	0.2251	0.1423	1.1428
-2.56	0.0151	0.0052	2.5617	-2.06	0.0478	0.0197	2.0672	-1.56	0.1182	0.0594	1.5855	-1.06	0.2275	0.1446	1.1342
-2.55	0.0154	0.0054	2.5517	-2.05	0.0488	0.0202	2.0574	-1.55	0.1200	0.0606	1.5761	-1.05	0.2299	0.1469	1.1257
-2.54	0.0158	0.0055	2.5418	-2.04	0.0498	0.0207	2.0476	-1.54	0.1219	0.0618	1.5667	-1.04	0.2323	0.1492	1.1172
-2.53	0.0163	0.0057	2.5318	-2.03	0.0508	0.0212	2.0378	-1.53	0.1238	0.0630	1.5574	-1.03	0.2347	0.1515	1.1087
-2.52	0.0167	0.0059	2.5219	-2.02	0.0519	0.0217	2.0280	-1.52	0.1257	0.0643	1.5480	-1.02	0.2371	0.1539	1.1002
-2.51	0.0171	0.0060	2.5119	-2.01	0.0529	0.0222	2.0183	-1.51	0.1276	0.0655	1.5386	-1.01	0.2396	0.1562	1.0917

z	f(z)	F(z)	L(z)	z	f(z)	F(z)	L(z)	z	f(z)	F(z)	L(z)	z	f(z)	F(z)	L(z)
-1.00	0.2420	0.1587	1.0833	-0.50	0.3521	0.3085	0.6978	0.00	0.3989	0.5000	0.3989	0.50	0.3521	0.6915	0.1978
-0.99	0.2444	0.1611	1.0749	-0.49	0.3538	0.3121	0.6909	0.01	0.3989	0.5040	0.3940	0.51	0.3503	0.6950	0.1947
-0.98	0.2468	0.1635	1.0665	-0.48	0.3555	0.3156	0.6840	0.02	0.3989	0.5080	0.3890	0.52	0.3485	0.6985	0.1917
-0.97	0.2492	0.1660	1.0582	-0.47	0.3572	0.3192	0.6772	0.03	0.3988	0.5120	0.3841	0.53	0.3467	0.7019	0.1887
-0.96	0.2516	0.1685	1.0499	-0.46	0.3589	0.3228	0.6704	0.04	0.3986	0.5160	0.3793	0.54	0.3448	0.7054	0.1857
-0.95	0.2541	0.1711	1.0416	-0.45	0.3605	0.3264	0.6637	0.05	0.3984	0.5199	0.3744	0.55	0.3429	0.7088	0.1828
-0.94	0.2565	0.1736	1.0333	-0.44	0.3621	0.3300	0.6569	0.06	0.3982	0.5239	0.3697	0.56	0.3410	0.7123	0.1799
-0.93	0.2589	0.1762	1.0250	-0.43	0.3637	0.3336	0.6503	0.07	0.3980	0.5279	0.3649	0.57	0.3391	0.7157	0.1771
-0.92	0.2613	0.1788	1.0168	-0.42	0.3653	0.3372	0.6436	0.08	0.3977	0.5319	0.3602	0.58	0.3372	0.7190	0.1742
-0.91	0.2637	0.1814	1.0086	-0.41	0.3668	0.3409	0.6370	0.09	0.3973	0.5359	0.3556	0.59	0.3352	0.7224	0.1714
-0.90	0.2661	0.1841	1.0004	-0.40	0.3683	0.3446	0.6304	0.10	0.3970	0.5398	0.3509	0.60	0.3332	0.7257	0.1687
-0.89	0.2685	0.1867	0.9923	-0.39	0.3697	0.3483	0.6239	0.11	0.3965	0.5438	0.3464	0.61	0.3312	0.7291	0.1659
-0.88	0.2709	0.1894	0.9842	-0.38	0.3712	0.3520	0.6174	0.12	0.3961	0.5478	0.3418	0.62	0.3292	0.7324	0.1633
-0.87	0.2732	0.1922	0.9761	-0.37	0.3725	0.3557	0.6109	0.13	0.3956	0.5517	0.3373	0.63	0.3271	0.7357	0.1606
-0.86	0.2756	0.1949	0.9680	-0.36	0.3739	0.3594	0.6045	0.14	0.3951	0.5557	0.3328	0.64	0.3251	0.7389	0.1580
-0.85	0.2780	0.1977	0.9600	-0.35	0.3752	0.3632	0.5981	0.15	0.3945	0.5596	0.3284	0.65	0.3230	0.7422	0.1554
-0.84	0.2803	0.2005	0.9520	-0.34	0.3765	0.3669	0.5918	0.16	0.3939	0.5636	0.3240	0.66	0.3209	0.7454	0.1528
-0.83	0.2827	0.2033	0.9440	-0.33	0.3778	0.3707	0.5855	0.17	0.3932	0.5675	0.3197	0.67	0.3187	0.7486	0.1503
-0.82	0.2850	0.2061	0.9360	-0.32	0.3790	0.3745	0.5792	0.18	0.3925	0.5714	0.3154	0.68	0.3166	0.7517	0.1478
-0.81	0.2874	0.2090	0.9281	-0.31	0.3802	0.3783	0.5730	0.19	0.3918	0.5753	0.3111	0.69	0.3144	0.7549	0.1453
-0.80	0.2897	0.2119	0.9202	-0.30	0.3814	0.3821	0.5668	0.20	0.3910	0.5793	0.3069	0.70	0.3123	0.7580	0.1429
-0.79	0.2920	0.2148	0.9123	-0.29	0.3825	0.3859	0.5606	0.21	0.3902	0.5832	0.3027	0.71	0.3101	0.7611	0.1405
-0.78	0.2943	0.2177	0.9045	-0.28	0.3836	0.3897	0.5545	0.22	0.3894	0.5871	0.2986	0.72	0.3079	0.7642	0.1381
-0.77	0.2966	0.2206	0.8967	-0.27	0.3847	0.3936	0.5484	0.23	0.3885	0.5910	0.2944	0.73	0.3056	0.7673	0.1358
-0.76	0.2989	0.2236	0.8889	-0.26	0.3857	0.3974	0.5424	0.24	0.3876	0.5948	0.2904	0.74	0.3034	0.7704	0.1334
-0.75	0.3011	0.2266	0.8812	-0.25	0.3867	0.4013	0.5363	0.25	0.3867	0.5987	0.2863	0.75	0.3011	0.7734	0.1312
-0.74	0.3034	0.2296	0.8734	-0.24	0.3876	0.4052	0.5304	0.26	0.3857	0.6026	0.2824	0.76	0.2989	0.7764	0.1289
-0.73	0.3056	0.2327	0.8658	-0.23	0.3885	0.4090	0.5244	0.27	0.3847	0.6064	0.2784	0.77	0.2966	0.7794	0.1267
-0.72	0.3079	0.2358	0.8581	-0.22	0.3894	0.4129	0.5186	0.28	0.3836	0.6103	0.2745	0.78	0.2943	0.7823	0.1245
-0.71	0.3101	0.2389	0.8505	-0.21	0.3902	0.4168	0.5127	0.29	0.3825	0.6141	0.2706	0.79	0.2920	0.7852	0.1223
-0.70	0.3123	0.2420	0.8429	-0.20	0.3910	0.4207	0.5069	0.30	0.3814	0.6179	0.2668	0.80	0.2897	0.7881	0.1202
-0.69	0.3144	0.2451	0.8353	-0.19	0.3918	0.4247	0.5011	0.31	0.3802	0.6217	0.2630	0.81	0.2874	0.7910	0.1181
-0.68	0.3166	0.2483	0.8278	-0.18	0.3925	0.4286	0.4954	0.32	0.3790	0.6255	0.2592	0.82	0.2850	0.7939	0.1160
-0.67	0.3187	0.2514	0.8203	-0.17	0.3932	0.4325	0.4897	0.33	0.3778	0.6293	0.2555	0.83	0.2827	0.7967	0.1140
-0.66	0.3209	0.2546	0.8128	-0.16	0.3939	0.4364	0.4840	0.34	0.3765	0.6331	0.2518	0.84	0.2803	0.7995	0.1120
-0.65	0.3230	0.2578	0.8054	-0.15	0.3945	0.4404	0.4784	0.35	0.3752	0.6368	0.2481	0.85	0.2780	0.8023	0.1100
-0.64	0.3251	0.2611	0.7980	-0.14	0.3951	0.4443	0.4728	0.36	0.3739	0.6406	0.2445	0.86	0.2756	0.8051	0.1080
-0.63	0.3271	0.2643	0.7906	-0.13	0.3956	0.4483	0.4673	0.37	0.3725	0.6443	0.2409	0.87	0.2732	0.8078	0.1061
-0.62	0.3292	0.2676	0.7833	-0.12	0.3961	0.4522	0.4618	0.38	0.3712	0.6480	0.2374	0.88	0.2709	0.8106	0.1042
-0.61	0.3312	0.2709	0.7759	-0.11	0.3965	0.4562	0.4564	0.39	0.3697	0.6517	0.2339	0.89	0.2685	0.8133	0.1023
-0.60	0.3332	0.2743	0.7687	-0.10	0.3970	0.4602	0.4509	0.40	0.3683	0.6554	0.2304	0.90	0.2661	0.8159	0.1004
-0.59	0.3352	0.2776	0.7614	-0.09	0.3973	0.4641	0.4456	0.41	0.3668	0.6591	0.2270	0.91	0.2637	0.8186	0.0986
-0.58	0.3372	0.2810	0.7542	-0.08	0.3977	0.4681	0.4402	0.42	0.3653	0.6628	0.2236	0.92	0.2613	0.8212	0.0968
-0.57	0.3391	0.2843	0.7471	-0.07	0.3980	0.4721	0.4349	0.43	0.3637	0.6664	0.2203	0.93	0.2589	0.8238	0.0950
-0.56	0.3410	0.2877	0.7399	-0.06	0.3982	0.4761	0.4297	0.44	0.3621	0.6700	0.2169	0.94	0.2565	0.8264	0.0933
-0.55	0.3429	0.2912	0.7328	-0.05	0.3984	0.4801	0.4244	0.45	0.3605	0.6736	0.2137	0.95	0.2541	0.8289	0.0916
-0.54	0.3448	0.2946	0.7257	-0.04	0.3986	0.4840	0.4193	0.46	0.3589	0.6772	0.2104	0.96	0.2516	0.8315	0.0899
-0.53	0.3467	0.2981	0.7187	-0.03	0.3988	0.4880	0.4141	0.47	0.3572	0.6808	0.2072	0.97	0.2492	0.8340	0.0882
-0.52	0.3485	0.3015	0.7117	-0.02	0.3989	0.4920	0.4090	0.48	0.3555	0.6844	0.2040	0.98	0.2468	0.8365	0.0865
-0.51	0.3503	0.3050	0.7047	-0.01	0.3989	0.4960	0.4040	0.49	0.3538	0.6879	0.2009	0.99	0.2444	0.8389	0.0849

z	f(z)	F(z)	L(z)	z	f(z)	F(z)	L(z)	z	f(z)	F(z)	L(z)	z	f(z)	F(z)	L(z)
1.00	0.2420	0.8413	0.0833	1.50	0.1295	0.9332	0.0293	2.00	0.0540	0.9772	0.0085	2.50	0.0175	0.9938	0.0020
1.01	0.2396	0.8438	0.0817	1.51	0.1276	0.9345	0.0286	2.01	0.0529	0.9778	0.0083	2.51	0.0171	0.9940	0.0019
1.02	0.2371	0.8461	0.0802	1.52	0.1257	0.9357	0.0280	2.02	0.0519	0.9783	0.0080	2.52	0.0167	0.9941	0.0019
1.03	0.2347	0.8485	0.0787	1.53	0.1238	0.9370	0.0274	2.03	0.0508	0.9788	0.0078	2.53	0.0163	0.9943	0.0018
1.04	0.2323	0.8508	0.0772	1.54	0.1219	0.9382	0.0267	2.04	0.0498	0.9793	0.0076	2.54	0.0158	0.9945	0.0018
1.05	0.2299	0.8531	0.0757	1.55	0.1200	0.9394	0.0261	2.05	0.0488	0.9798	0.0074	2.55	0.0154	0.9946	0.0017
1.06	0.2275	0.8554	0.0742	1.56	0.1182	0.9406	0.0255	2.06	0.0478	0.9803	0.0072	2.56	0.0151	0.9948	0.0017
1.07	0.2251	0.8577	0.0728	1.57	0.1163	0.9418	0.0249	2.07	0.0468	0.9808	0.0070	2.57	0.0147	0.9949	0.0016
1.08	0.2227	0.8599	0.0714	1.58	0.1145	0.9429	0.0244	2.08	0.0459	0.9812	0.0068	2.58	0.0143	0.9951	0.0016
1.09	0.2203	0.8621	0.0700	1.59	0.1127	0.9441	0.0238	2.09	0.0449	0.9817	0.0066	2.59	0.0139	0.9952	0.0015
1.10	0.2179	0.8643	0.0686	1.60	0.1109	0.9452	0.0232	2.10	0.0440	0.9821	0.0065	2.60	0.0136	0.9953	0.0015
1.11	0.2155	0.8665	0.0673	1.61	0.1092	0.9463	0.0227	2.11	0.0431	0.9826	0.0063	2.61	0.0132	0.9955	0.0014
1.12	0.2131	0.8686	0.0659	1.62	0.1074	0.9474	0.0222	2.12	0.0422	0.9830	0.0061	2.62	0.0129	0.9956	0.0014
1.13	0.2107	0.8708	0.0646	1.63	0.1057	0.9484	0.0216	2.13	0.0413	0.9834	0.0060	2.63	0.0126	0.9957	0.0013
1.14	0.2083	0.8729	0.0634	1.64	0.1040	0.9495	0.0211	2.14	0.0404	0.9838	0.0058	2.64	0.0122	0.9959	0.0013
1.15	0.2059	0.8749	0.0621	1.65	0.1023	0.9505	0.0206	2.15	0.0396	0.9842	0.0056	2.65	0.0119	0.9960	0.0012
1.16	0.2036	0.8770	0.0609	1.66	0.1006	0.9515	0.0201	2.16	0.0387	0.9846	0.0055	2.66	0.0116	0.9961	0.0012
1.17	0.2012	0.8790	0.0596	1.67	0.0989	0.9525	0.0197	2.17	0.0379	0.9850	0.0053	2.67	0.0113	0.9962	0.0012
1.18	0.1989	0.8810	0.0584	1.68	0.0973	0.9535	0.0192	2.18	0.0371	0.9854	0.0052	2.68	0.0110	0.9963	0.0011
1.19	0.1965	0.8830	0.0573	1.69	0.0957	0.9545	0.0187	2.19	0.0363	0.9857	0.0050	2.69	0.0107	0.9964	0.0011
1.20	0.1942	0.8849	0.0561	1.70	0.0940	0.9554	0.0183	2.20	0.0355	0.9861	0.0049	2.70	0.0104	0.9965	0.0011
1.21	0.1919	0.8869	0.0550	1.71	0.0925	0.9564	0.0178	2.21	0.0347	0.9864	0.0047	2.71	0.0101	0.9966	0.0010
1.22	0.1895	0.8888	0.0538	1.72	0.0909	0.9573	0.0174	2.22	0.0339	0.9868	0.0046	2.72	0.0099	0.9967	0.0010
1.23	0.1872	0.8907	0.0527	1.73	0.0893	0.9582	0.0170	2.23	0.0332	0.9871	0.0045	2.73	0.0096	0.9968	0.0010
1.24	0.1849	0.8925	0.0517	1.74	0.0878	0.9591	0.0166	2.24	0.0325	0.9875	0.0044	2.74	0.0093	0.9969	0.0009
1.25	0.1826	0.8944	0.0506	1.75	0.0863	0.9599	0.0162	2.25	0.0317	0.9878	0.0042	2.75	0.0091	0.9970	0.0009
1.26	0.1804	0.8962	0.0495	1.76	0.0848	0.9608	0.0158	2.26	0.0310	0.9881	0.0041	2.76	0.0088	0.9971	0.0009
1.27	0.1781	0.8980	0.0485	1.77	0.0833	0.9616	0.0154	2.27	0.0303	0.9884	0.0040	2.77	0.0086	0.9972	0.0008
1.28	0.1758	0.8997	0.0475	1.78	0.0818	0.9625	0.0150	2.28	0.0297	0.9887	0.0039	2.78	0.0084	0.9973	0.0008
1.29	0.1736	0.9015	0.0465	1.79	0.0804	0.9633	0.0146	2.29	0.0290	0.9890	0.0038	2.79	0.0081	0.9974	0.0008
1.30	0.1714	0.9032	0.0455	1.80	0.0790	0.9641	0.0143	2.30	0.0283	0.9893	0.0037	2.80	0.0079	0.9974	0.0008
1.31	0.1691	0.9049	0.0446	1.81	0.0775	0.9649	0.0139	2.31	0.0277	0.9896	0.0036	2.81	0.0077	0.9975	0.0007
1.32	0.1669	0.9066	0.0436	1.82	0.0761	0.9656	0.0136	2.32	0.0270	0.9898	0.0035	2.82	0.0075	0.9976	0.0007
1.33	0.1647	0.9082	0.0427	1.83	0.0748	0.9664	0.0132	2.33	0.0264	0.9901	0.0034	2.83	0.0073	0.9977	0.0007
1.34	0.1626	0.9099	0.0418	1.84	0.0734	0.9671	0.0129	2.34	0.0258	0.9904	0.0033	2.84	0.0071	0.9977	0.0007
1.35	0.1604	0.9115	0.0409	1.85	0.0721	0.9678	0.0126	2.35	0.0252	0.9906	0.0032	2.85	0.0069	0.9978	0.0006
1.36	0.1582	0.9131	0.0400	1.86	0.0707	0.9686	0.0123	2.36	0.0246	0.9909	0.0031	2.86	0.0067	0.9979	0.0006
1.37	0.1561	0.9147	0.0392	1.87	0.0694	0.9693	0.0119	2.37	0.0241	0.9911	0.0030	2.87	0.0065	0.9979	0.0006
1.38	0.1539	0.9162	0.0383	1.88	0.0681	0.9699	0.0116	2.38	0.0235	0.9913	0.0029	2.88	0.0063	0.9980	0.0006
1.39	0.1518	0.9177	0.0375	1.89	0.0669	0.9706	0.0113	2.39	0.0229	0.9916	0.0028	2.89	0.0061	0.9981	0.0006
1.40	0.1497	0.9192	0.0367	1.90	0.0656	0.9713	0.0111	2.40	0.0224	0.9918	0.0027	2.90	0.0060	0.9981	0.0005
1.41	0.1476	0.9207	0.0359	1.91	0.0644	0.9719	0.0108	2.41	0.0219	0.9920	0.0026	2.91	0.0058	0.9982	0.0005
1.42	0.1456	0.9222	0.0351	1.92	0.0632	0.9726	0.0105	2.42	0.0213	0.9922	0.0026	2.92	0.0056	0.9982	0.0005
1.43	0.1435	0.9236	0.0343	1.93	0.0620	0.9732	0.0102	2.43	0.0208	0.9925	0.0025	2.93	0.0055	0.9983	0.0005
1.44	0.1415	0.9251	0.0336	1.94	0.0608	0.9738	0.0100	2.44	0.0203	0.9927	0.0024	2.94	0.0053	0.9984	0.0005
1.45	0.1394	0.9265	0.0328	1.95	0.0596	0.9744	0.0097	2.45	0.0198	0.9929	0.0023	2.95	0.0051	0.9984	0.0005
1.46	0.1374	0.9279	0.0321	1.96	0.0584	0.9750	0.0094	2.46	0.0194	0.9931	0.0023	2.96	0.0050	0.9985	0.0004
1.47	0.1354	0.9292	0.0314	1.97	0.0573	0.9756	0.0092	2.47	0.0189	0.9933	0.0022	2.97	0.0048	0.9985	0.0004
1.48	0.1334	0.9306	0.0307	1.98	0.0562	0.9761	0.0090	2.48	0.0184	0.9934	0.0021	2.98	0.0047	0.9986	0.0004
1.49	0.1315	0.9319	0.0300	1.99	0.0551	0.9767	0.0087	2.49	0.0180	0.9936	0.0021	2.99	0.0046	0.9986	0.0004