Universität zu Köln

Seminar für Supply Chain Management und Management Science

Primer in Inventory Management - Inventory Optimization Project

You are in charge of managing inventory at a high-end electronics retailer. The retailer sells over 100,000 different parts. In a project, three typical parts were selected and your task is to answer various questions that the retailer has.

Part DP (10 points)

Part DP is an ultra-resolution display that is used in navigation systems for aircrafts and ships. The retailer stocks the product and demands that come in are typically spare-part demands from customers who must replace a failed part. If the retailer cannot fill a customer demand, the demand is backordered and the aircraft or ship remains grounded. The cost of a grounded aircraft or ship is quite high. Because of the high criticality of the part, the customers and the retailer have agreed on a contract where the retailer must pay a backorder penalty cost 1,000 EUR/day that the demand is backordered.

The display sells for 6,000 EUR/unit. The retailer uses an opportunity cost of capital of 20 % per year. The display is bought from a supplier at a price of 5,000 EUR/unit. The lead time is seven days. The display is stored in a high protection area that is leased from a third party logistics provider at 100 EUR per square meter per month. Each unit requires half a square meter of storage.

The retailer uses an order-up-to level inventory policy with an order-up-to level and re-orders each day. The retailer wonders, if the current order-up-to level is optimal.

1. For comparison, compute the optimal solution:
   a. The retailer has provided demand data from the last 13 months. Plot the data and fit an appropriate distribution to the data. Estimate the parameter of the distribution.
   b. Determine the critical ratio of the part.
   c. Determine the optimal order quantity of the part.
   d. Determine the expected cost and expected profit of the optimal solution.

2. The retailer is currently using as a rule of thumb the following formula, that somebody suggested would work well:

$$S = \frac{\text{annual demand}}{365}\text{lead time} + 2\sqrt{\text{lead time}}\ \text{standard deviation of daily demand}$$

   a. What could be the rationale behind this formula?
   b. Does the formula contain the basic elements that are relevant for inventory management?
   c. Apply the formula using the demand data from the year 2011 and round the solution to the closest integer.
   d. What is the expected cost of the solution?
   e. How much can the retailer increase her annual profit if she uses the optimal solution as opposed to the rule-of-thumb solution?

Part DN (10 points)

Part DN is a cable for a power supply. The retailer stocks the product and excess demand is backordered. If a demand is backordered, the retailer informs the customers about the backorder. 50 % of the customer get back to the retailer and inquire about the backorder. A service center responds to the inquiries and charges 10 minutes at 1.00 EUR/minute for the responses. Therefore, the retailer uses unit backorder penalty cost of 5.00 EUR/backorder.

The retailer sells the cable at 2.99 EUR/unit. The retailer uses an opportunity cost of capital of 20 % per year. The supplier is located in Asia and has a lead time of 45 days. The supplier charges a price of 0.20 EUR/unit. The supplier charges 0.01 EUR/unit for shipping, plus a fixed order charge of 20 EUR/order. The fixed order cost at the retailer is 40 EUR/order.

The retailer uses a continuous review inventory policy and places an order immediately, if the inventory level reaches r. The retailer wonders, if the current re-order point and order quantity are optimal.

1. For comparison, compute the optimal solution:
    a. The retailer has provided demand data from the last 13 months. Plot the data and fit an appropriate distribution to the data. Estimate the parameters of the distribution.
    b. Determine the optimal re-order point and the optimal order quantity.
    c. Determine the expected cost of the optimal solution.

2. The retailer is currently using as a rule of thumb the following formulae, that somebody suggested would work well:

$$x^* = \sqrt{\frac{2\mu K}{h}}$$

$$r^* = \mu_{LT} + 2\sigma_{LT}$$

    a. What could be the rationale behind this formula?
    b. Does the formula contain the basic elements that are relevant for inventory management? If not, what is missing?
    c. Apply the formula using the demand data you have and round the solution to the closest integer.
    d. What is the expected cost of the solution?
    e. How much can the retailer increase her annual profit if she uses the optimal solution as opposed to the rule-of-thumb solution?

3. With the optimal solution, which α-service-level does the retailer achieve?

4. With the optimal solution, which β-service-level does the retailer achieve?

5. If the retailer wanted to require β-service-level of 98 %, what would be the optimal solution?

Part FF (10 points)

Part FF is a power supply. It is often demanded jointly with the cable (Part DN), which explains that the demands are identical. For the analysis, we do not take this correlation into account. The inventory is managed using an order-up-to level inventory policy with an α-service-level of α = 95.0 %. The lead time is two days and orders are placed daily.

1. The retailer wants to use Moving Averages to forecast the demand of the power supply and is wondering over how many periods she should average. She asks you to "evaluate the performance of the forecasting approach" for K = 5, 50, 100, and 200 periods (K is the number of periods over which we average).
   You look up performance measure for forecasting algorithms in a textbook and find the following two:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(\underbrace{\hat{y}_t - y_t}_{\varepsilon_t})^2 \qquad SE = \frac{1}{T}\sum_{t=1}^{T}\underbrace{(\hat{y}_t - y_t)}_{\varepsilon_t}$$

   a. What do these performance measures measure?

   b. Plot the demand data.

   c. Compute the values of the performance measures for K = 5, 50, 100, and 200 (or at a higher granularity, if you wish) and select a good value for K.

   d. To compute the optimal order-up-to level, we need an estimate of the standard deviation of the forecast error. Use the data points from periods t – M to t – 1 to estimate the forecast error in period t. Estimate the standard deviation of the forecast error for period K + M + 1. Use M = 30 for your computations.

      Note that you need periods 1 to K to forecast demand for period K + 1. So, the first forecast error that you estimate is the forecast error in period K + 1. Because you average over M periods to estimate the forecast error, you need the forecast errors of periods K + 1 to K + M for your first estimate. Therefore, the first period for which you can compute an order-up-to level is Period K + M + 1.

   e. Compute the order-up-to level S_t from Period K + M + 1 until Feb. 1, 2012.

   f. For each period, determine
      i. I_t, the inventory at the beginning (= I^{end}_{t-1})
      ii. O_t, the number of outstanding orders
      iii. $IP_t = I_t + O_t$, the inventory position
      iv. $x_t = S – IP_t$, the order quantity
      v. x_{t-LT}, the order that was placed LT periods earlier and arrives in period t
      vi. $I^{end}_t = I_t + x_{t-LT} – y_t$, the inventory at the end of period t (y_t is the demand in t)

Your spreadsheet could look similar to the following one:

Period	Date	Demand	yhat_t	sigmahat_t	Sstar	1. I_t	2. O_t	3. IP_t	4. x_t	5. x_t-LT	I end of t
132	5/12/2011	70	106.0	37.4364768	424	-17	273	256	168	148	61
133	5/13/2011	62	105.5	37.5874163	423	61	293	354	69	125	124
134	5/14/2011	150	104.8	38.3288842	423	124	237	361	62	168	142
135	5/15/2011	70	104.6	39.0604393	425	142	131	273	152	69	141
136	5/16/2011	46	104.6	38.6082621	423	141	214	355	68	62	157
137	5/17/2011	132	102.8	39.0733937	419	157	220	377	42	152	177
138	5/18/2011	72	103.1	39.2754144	421	177	110	287	134	68	173
139	5/19/2011	78	103.8	39.6952439	424	173	176	349	75	42	137
140	5/20/2011	92	102.9	38.7877476	419	137	209	346	73	134	179
141	5/21/2011	117	103.0	38.6802815	419	179	148	327	92	75	137
142	5/22/2011	102	103.9	38.6110461	421	137	165	302	119	73	108
143	5/23/2011	58	104.5	38.3151768	422	108	211	319	103	92	142
144	5/24/2011	96	103.3	39.2359464	421	142	222	364	57	119	165
145	5/25/2011	130	102.8	36.335606	412	165	160	325	87	103	138
146	5/26/2011	104	103.3	36.4764527	414	138	144	282	132	57	91
147	5/27/2011	59	103.0	35.1373518	409	91	219	310	99	87	119
148	5/28/2011	172	102.5	35.6561845	409	119	231	350	59	132	79
149	5/29/2011	110	103.0	36.5727206	413	79	158	237	176	99	68
150	5/30/2011	112	102.6	36.5097261	412	68	235	303	109	59	15

Part FT (10 points)

Part FF is a battery that has twice the capacity of comparable products. The battery was introduced quite recently and it is gaining popularity. Its lead time is five days and inventory is reviewed daily. Because the price of the battery depletes quickly (about 2 % per week), the retailers uses a moderate service level of α = 90 %. The retailer has been quite satisfied with your work so far and asks you to optimize the order-up-to level of the part.

She hints that there seems to be a trend in the demand that should **not** be neglected. Then she asks you to recall that the order-up-to level in period t must be set, such that the probability of a stock out in period t + LT is α = 90 %. Finally, she reminds you, that the demand over lead time plus one period is the relevant demand. Before she closes the door to leave you with your assignment, she suggests that you conduct an analysis similar to the one that you conducted for Part FF. So you plot the demand data, forecast demand with a suiting forecasting algorithm, estimate the standard deviation of the forecast error, compute the order-up-to-level and determine relevant values as in f.

Parts MX (40 points)

Parts MX1 – MX4 are four products with varying demand characteristics. Holding costs are 1 Euro/unit/week, penalty costs are 9 Euro/unit/week. Orders are placed daily. Inventory is controlled by a periodic review policy. The lead time is four days. Based on the demand data, the retailer asks you to come up with a formula for order quantities for each product that she can apply to future demands. You can fit a theoretical demand distribution to the data or use a forecasting method. You can also use different approaches for different parts. The objective is to minimize costs in future periods.

For this task, you are given a table in sheet MX. Please fill in formulas (or values) for **each** yellow cell (D2-D1000, F2-F1000, H2-H1000, J2-J1000) and the ending inventory of period 399 in the yellow table (L2-M6). **You may add additional fields for your own computations, but do not write down your solutions elsewhere. We will grade you based upon the values in the yellow cells.**
There are yellow cells in rows without demand, so that your formula will yield unreasonable results for now. We will insert demand values later and check how well your formula performs. Please hand in your filled table with your other solutions in ILIAS. Future demands will be generated by the same process as historical demands and you will be evaluated by the total realized cost over periods 400 – 999.