Primer on Inventory Management – Excercises

Economic Order Quantity Model

Exercise 1: Solution of the EOQ model

In the video, I explain that the cost function is convex in the order quantity x, because fixed order cost and inventory holding cost are convex in the order quantity x and the sum of convex functions is convex. An alternative proof is to show that the second derivative of the cost function is positive.

i. Please show that the second derivative of $Z(x) = h\frac{x}{2} + K\frac{\mu}{x} + c\mu$ is always positive.

ii. Set the first derivative equal to zero and solve for $x^\star$.

iii. Use the value of $x^\star$ in $Z(x)$ to compute the cost of the optimal solution $Z(x^\star)$.

iv. Show that the optimal fixed order cost have the same value as the optimal inventory holding cost.

Exercise 2: Application of the EOQ model

Pencils at the campus bookstore are sold at a steady rate of 60 per week. The bookstore can purchase the pencils for 2 cents each and sells them at a price of 15 cents each. It costs the bookstore 12 EUR to initiate an order, and holding cost are based on an annual interest rate of 25 percent.

i. Determine the optimal order quantity $x^\star$.

ii. Determine the cost of the optimal solution $Z(x^\star)$.

Exercise 3: Sensititvity of the solution

In this exercise, you will analyze how the cost of a solution is affected if we deviate from the optimal solution. For example, if the optimal order quantity were $x^\star = 92$ and we would round up the order quantity to $x = 100$, e.g. the package size could be 100, and therefore $x = 100$ would be preferable. Then an insensitive solution would have about the same cost for $x = 100$ than for $x^\star = 92$. An insensitive solution is desirable, because it allows us to deviate from the optimal solution without major increases in cost.

i. Assume that $K = 40$, $h = 0.1$, $c = 1$, and $\mu = 10.6$. Compute the optimal order quantity $x^\star$.

ii. Compute the optimal cost $Z\left(x^\star\right)$.

iii. Assume that you want to order 10 % less than $x^\star$, i.e., $x = 0.90x^\star$, how much would cost increase with this order quantity?

iv. Derive a general expression for the sensitivity of solution, by deriving a formula for $\mathcal{Z}\left(x\right)/\mathcal{Z}\left(x^\star\right)$, where $\mathcal{Z}\left(x\right) = Z\left(x\right) - c\mu$.

v. Plot $\mathcal{Z}\left(x\right)/\mathcal{Z}\left(x^\star\right)$ for $x = a \cdot x^\star$ with $a = 0.5, 0.51, \ldots, 1.5$.

vi. How do you evaluate the sensitivity of the solution?